immediately preceding the input edge of the blade; Tu, local value of the degree of turbulence at the surface; Re_0, Reynolds number of incoming flow in terms of the parameters preceding the turbine assembly.

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STARTUP AND STEADY OPERATING CONDITIONS OF SUPERSONIC DIFFUSER

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An engineering method is proposed for calculating the gas dynamics, friction, and heat transfer of steady and nonsteady flows in supersonic diffusers. The calculation results are compared with experimental data for theoretical Mach numbers M = 2 and 3 preceding the diffuser.

1. Introduction

An urgent problem in increasing the operational efficiency of power plants is the creation of a supersonic diffuser with reduced total-pressure losses in operating and variable conditions. Decrease in the startup pressure difference is especially important, since in real diffusers with uncontrollable geometry the total-pressure losses in startup conditions are at least twice the losses in operating conditions [1]. The actual flow pattern in the startup of a supersonic diffuser is very complex and depends significantly on the startup method, channel geometry, the methods used to control the boundary layer and the heat-transfer shielding of the walls, and a number of other factors. It is necessary to create a reliable engineering method of calculating the gas-flow parameters in supersonic diffusers since the application of shaped diffusers permits significant reduction in total-pressure loss [2].

In creating an engineering method, it is necessary to solve the problem of calculating the nonsteady turbulent boundary layer, taking account of perturbing factors such as the compressibility, nonisothermal conditions, significant positive longitudinal pressure gradient, surface roughness, interaction with density discontinuities, and flow singularities due to the methods used to control the boundary layer and the heat-transfer shielding of the walls.

Experimental data on the startup and steady operating conditions of a supersonic diffuser are outlined below, and an engineering method of calculating the gas dynamics, friction, and heat transfer in gas flow in a supersonic diffuser is proposed.

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2. Mathematical Model

To describe the flow in a supersonic diffuser, it is divided into two regions: the flow core with homogeneous parameters and the boundary layer [3]. For a turbulent nonsteady boundary layer, it is assumed that:

1) the averaging period of the parameters is chosen to be sufficiently large in comparison with the time scale of turbulence and sufficiently small in comparison with the period of flow variation over time unrelated to turbulence;

2) the nonsteady behavior of the mean turbulent flow has no direct influence on the boundary-layer structure (quasi-steady boundary layer), i.e., the basic hypotheses of semi-empirical theory are adopted [3];

3) there is analogy between the influence of the longitudinal pressure gradient and nonsteady conditions [3];

4) the flow characteristics at corners may be neglected.

To calculate a turbulent boundary layer, the relative frictional law must be known with the simultaneous action of several perturbing factors. In [4], the use of the superposition principle was proposed, and the combined influence of these factors was taken into account in the critical values of the corresponding form parameters. However, this requires verification. To that end, the limiting relative frictional law is obtained analytically, taking account of the combined action of the longitudinal pressure gradient, nonsteady conditions, compressibility, and nonisothermal conditions by means of the asymptotic theory of a turbulent boundary layer [4], using the Van Dyck perturbation method.

As $\mathrm{Re} \to \infty$

$$\Psi_\Sigma = \left[\frac{1 - \sqrt{\bar{f}_\delta}}{\int\limits_0^1 \frac{d\omega}{\tilde{\rho}}} \right]^2, \tag{1}$$

where

$$\bar{f}_\delta = \frac{f_\delta}{f_{\delta\,\mathrm{cr}}}, \quad \sqrt{-f_{\delta\,\mathrm{cr}}} = \frac{1}{\int\limits_0^1 \sqrt{\frac{\varphi}{\tilde{\rho}}\left(-\frac{dD_0}{d\xi}\right)}\, d\xi}.$$

For a steady compressible nonisothermal turbulent boundary layer with a positive longitudinal pressure gradient, at Mach numbers M = 0-5 and $\bar{\Psi}$ = 0.2-1.5, the limiting relative frictional law in Eq. (1) coincides with the frictional law determined from the superposition law for perturbing factors ($\Psi_\Sigma = \Psi_T \Psi_M \Psi_f$), to within 8% [4].

The closed system of equations for calculating the gas dynamics, friction, and heat transfer in a diffuser may be written in the form

$$\frac{\partial}{\partial t}(\rho_e F_e) + \frac{\partial}{\partial x}(\rho_e u_e F_e) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_e u_e F_e) + \frac{\partial}{\partial x}(\rho_e u_e^2 F_e + F_e p_e) = p_e \frac{\partial F_e}{\partial x}, \tag{3}$$

$$\frac{\partial}{\partial t}\left[\rho_e F_e\left(e_e + \frac{u_e^2}{2}\right)\right] + \frac{\partial}{\partial x}\left[\rho_e u_e F_e\left(e_e + \frac{p_e}{\rho_e} + \frac{u_e^2}{2}\right)\right] = 0, \tag{4}$$

$$\frac{\partial \mathrm{Re}^{**}}{\partial \bar{x}} + \mathrm{Re}_L (1 + H) f - \bar{j}_w \mathrm{Re}_L + \frac{L}{\mu_e} \frac{\partial}{\partial t}\left[\int\limits_0^\delta (\rho - \rho_e)\, dy\right] + \frac{L}{u_e}\frac{\partial}{\partial t}(\mathrm{Re}^{**} H) = \mathrm{Re}_L \frac{c_f}{2}, \tag{5}$$

$$\frac{\mathrm{Re}_L}{\rho_e u_e (T_w^* - T_w)} \frac{\partial}{\partial t}\left[T_e \int\limits_0^{\delta_T} (\rho - \rho_e)\, dy + \frac{u_e^2 \rho_e}{2c_p}\int\limits_0^{\delta_T} \tilde{\rho}(1 - \omega^2)\, dy\right] + \frac{\partial \mathrm{Re}_T^{**}}{\partial \bar{x}} + \mathrm{Re}_T^{**} f_t - \frac{\mathrm{Re}_L}{u_e (T_w^* - T_w)} \int\limits_0^{\delta_T} \tilde{\rho} dy \frac{\partial T_e^*}{\partial t} +$$

$$+\frac{\delta_T Re_L}{c_p\rho_e u_e (T_w^* - T_w)}\frac{\partial p_e}{\partial t} + Re_L \frac{\delta^* - \delta_T}{T_w^* - T_w}\frac{\partial T_e^*}{\partial x} - \bar{j}_w Re_L = St\, Re_L, \quad (6)$$

$$F_e = F - F^*, \quad (7)$$

where

$$c_f = c_{f0}\Psi_\Sigma \ [4]; \quad c_{f0} = \frac{B}{Re_w^{**m}} \ [4]; \quad \Psi_\Sigma = \Psi_M \Psi_T \Psi_{\lambda 0\Sigma}\Psi_{in}\Psi_R \Psi_{di};$$

$$St = St_0 \Psi_{s\Sigma} \ [4], \quad St_0 = \frac{B}{Re_{Tw}^{**m}}; \quad \Psi_{s\Sigma} = \Psi_M \Psi_T \Psi_{sR}\Psi_{sin}\Psi_{sh}\Psi_{sdi},$$

$$\tilde{\rho} = \frac{1}{\psi - \Delta\psi\omega\varepsilon - (\psi^* - 1)\omega^2} \ [4],$$

$H = H_0\bar{\Psi}(1.67\psi^* - 0.67)(1 + 0.1\lambda_{0\Sigma})$ [6]; Ψ_M and Ψ_T are taken from [4]; $\Psi_{\lambda 0}$, Ψ_{in}, Ψ_R, Ψ_{sin}, Ψ_{sh} from [6]; and Ψ_{sdi} from [7]. The difference between $\Psi_{\lambda 0\Sigma}$ and $\Psi_{\lambda 0}$ consists in the replacement of λ_0 by $\lambda_{0\Sigma}$. The nonsteady integral momentum and energy relations in Eqs. (5) and (6) are obtained by integrating the differential equations of the corresponding conservation laws [3].

In the interaction of the pressure discontinuity with the boundary layer, $u_e\partial p_e/\partial x \to \infty$. Using the analogy between the action of a longitudinal pressure gradient and nonsteady conditions and taking into account that $(\Psi_{\lambda 0\Sigma})_{turb} = 1.14$ and $(\Psi_{\lambda 0\Sigma})_{lam} = 1.65$ when $\partial p_e/\partial t \to \infty$ [8], the following relative frictional law is obtained for nonbreakaway interaction of the pressure discontinuity with the laminar and turbulent boundary layers: $(\Psi_{di})_{lam} = 1.65$; $(\Psi_{di})_{turb} = 1.14$. The relation between the equations describing the flow in the core and in the boundary layer is obtained using Eq. (7).

The velocity profile over the cross section of the turbulent boundary layer as $Re \to \infty$ is found from the equation in [4]

$$\int_\omega^1 \tilde{\rho}\, d\omega = \sqrt{\Psi_\Sigma}\int_{\omega_0}^1 \sqrt{\frac{\tilde{\tau}}{\tilde{\tau}_0}}\, d\omega_0. \quad (8)$$

The velocity profile at finite Reynolds numbers is determined by numerical solution of Eq. (8), taking account of the finiteness of the Reynolds number in Ψ_Σ.

In calculating the flow in a supersonic diffuser, it is assumed that the closing direct pressure discontinuity is at the point of boundary-layer breakaway due to the positive longitudinal pressure gradient and nonsteady conditions of the flow or the appearance of elevated pressure at the diffuser output upstream through the subsonic part of the boundary layer. Breakaway of the turbulent steady and nonsteady boundary layers is characterized by a critical parameter value [6]

$$\lambda_{0\Sigma cr} = 5.3\frac{1.43/\psi^{*0.72}}{1 + 0.43\psi^{0.9}/\psi^{*0.72}},$$

obtained by analysis of the most reliable experimental data. For the breakaway point due to penetration of the increased pressure beyond the diffuser into the channel, the parameter $\lambda_{0n} = (\delta^{**}/\tau_{w0})d(p_n - p_e)/dx$ is introduced; at the breakaway point, it is equal to $\lambda_{0\Sigma cr}$.

The system of equations for the flow core is solved numerically by the Godunov method [5]. The integral momentum and energy relations in Eqs. (5) and (6) are solved numerically by means of an explicit predictor—corrector scheme of second-order accuracy (in the first stage, by the Lax scheme [9] and in the second by the cross scheme [9]).

Test calculations of a standard turbulent boundary layer [4] and a Laval nozzle without taking account of the boundary layer show that the error in determining the flow parameters in these cases is no more than 1% in comparison with the anaytical solutions [4, 10].

3. Experimental Apparatus

The experimental apparatus consists of a continuous-action supersonic aerodynamic tube. Air from the compressor is fed along a pipeline through a baffle to the receiver and then

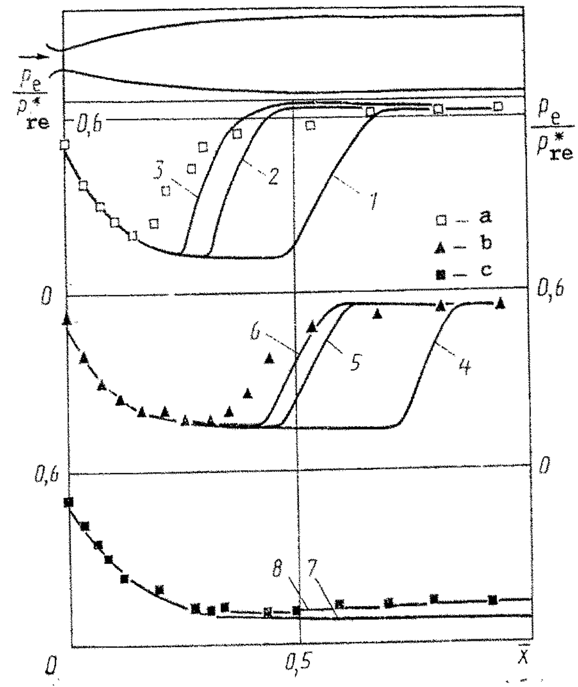

Fig. 1. Static-pressure distribution over the length of a supersonic diffuser of constant cross section with a theoretical Mach number M = 2 prior to the diffuser: a) p_{re}^* = 0.165 MPa, T_{re}^* = 320 K, $\bar{\psi}$ = 1, b) 0.176, 323, 1; c) 0.255, 328, 1. Calculation: 1, 4, 7) with no boundary layer; 2, 5) with a boundary layer but with no criterion of the position of the direct pressure discontinuity in the channel; 3, 6, 8) by the method here proposed.

to the Laval nozzle, the supersonic diffuser, and, through a muffler, to the atmosphere. The Laval nozzle assembly allows theoretical Mach numbers of M = 2 and 3 to be obtained in working conditions.

The nozzle length is 0.6 m; the length of the supersonic diffuser is 0.96 m. The output cross section of the nozzle is of dimensions 0.06 × 0.06 m. The lower wall of the supersonic diffuser is adjustable. Diffuser startup is by gradual opening of the baffle upstream from the receiver.

In the experiment with steady flow, the following parameters are measured:

the stagnation pressure and temperature in the receiver by means of a sampling manometer and a probe with a Chromel—Copel thermocouple, respectively;

the distribution of static pressure at the upper walls of the nozzle and the diffuser by means of a GRM-2 group recording manometer with an error no greater than 0.5% at the upper limit of measurement;

the total-pressure distribution over the diffuser cross section by means of a total-pressure microadapter connected to a micrometer. The error in determining $\omega = f(\xi)$ for an incompressible isothermal turbulent boundary layer at a smooth impermeable plate is no more than 2% in comparison with $\omega_0 = \xi^{1/7}$ (at Re** < 10^4) [4];

the upper-wall temperature of the diffuser in three cross sections over its length by means of Chromel—Copel thermocouples.

In investigating startup conditions, the total pressure in front of the Laval nozzle and the static-pressure distribution over the channel length are measured using a 10-channel information—measurement complex (IMC) with DMI pressure sensors. The error in measuring the nonsteady pressure values as a result of IMC metrological investigations in the range 0-2 kHz is no more than 10% of the upper limit.

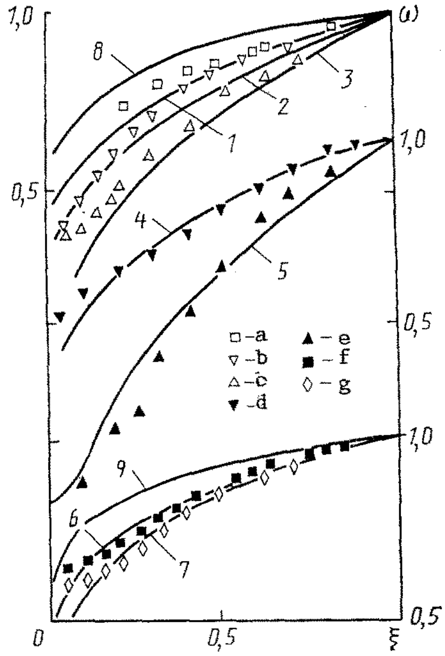

Fig. 2

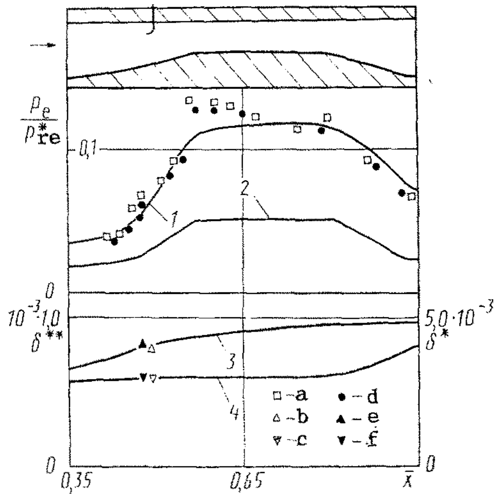

Fig. 3

Fig. 2. Distribution of the mean-velocity profile over the cross section of the turbulent boundary layer: when $Re^{**} < 10^4$: a) $f(Re^{**})^{0.25} = -9.06 \cdot 10^{-3}$ [4]; 1) calculation; b) $f(Re^{**})^{0.25} = -28.2 \cdot 10^{-3}$ [4]; 2) calculation; c) $f(Re^{**})^{0.25} = f(Re^{**})_{cr}^{0.25}$ [4]; 3) calculation; d) $M = 1.54$, $\lambda_0 = 1.1$; 4) calculation; e) $M = 2.5$, $\lambda_0 = \lambda_{0\Sigma cr}$ [12]; 5) calculation. When $Re^{**} > 10^4$: f) $M = 2.04$, $\lambda_0 = 0.66$; 6) calculation; g) $M = 2.08$, $\lambda_0 = 1.24$; 7) calculation; 8) $\omega = \xi^{1/7}$; 9) $\omega = \xi^{1/10}$.

Fig. 3. Distribution of static pressure and thicknesses of displacement and momentum loss over the length of a supersonic diffuser of variable cross section with theoretical Mach number $M = 3$ ($\bar{\psi} = 1$) preceding the diffuser. When $p_{re}^* = 0.54$ MPa, $T_{re}^* = 343$ K: a) p; 1) calculation by the proposed method; 2) calculation without boundary layer; b) δ^{**}; 3) calculation; c) δ^*; 4) calculation. When $p_{re}^* = 0.45$ MPa, $T_{re}^* = 338$ K: d) p; e) δ^{**}; f) δ^* (the theoretical distributions of the parameters practically coincide with the corresponding distributions of the first flow conditions). δ^*, δ^{**}, m.

4. Calculation of Flow Parameters

The results of comparing the theoretical static-pressure distribution over the length of a supersonic diffuser of constant cross section with a Laval nozzle (theoretical number $M = 2$) with experimental data are shown in Fig. 1.

Here and below, calculation is undertaken from the neck of the nozzle to the output cross section of the channel. The boundary condition at the neck of the nozzle for the boundary layer takes the form: $\delta = \delta^* = \delta^{**} = 0$.

The theoretical velocity profiles over the cross section of an incompressible and compressible steady isothermal turbulent boundary layer at an impermeable surface of small curvature with signifìcnat positive longitudinal pressure gradient obtained by numerical integration of Eq. (8) are compared with experimental data from the present work and the literature in Fig. 2.

Theoretical and experimental data for a working section with a Laval nozzle when $M = 3$ are compared in Fig. 3 for compressible isothermal flow with significant positive longitudinal pressure gradient and with the interaction of the turbulent boundary layer with low-intensity oblique pressure discontinuities.

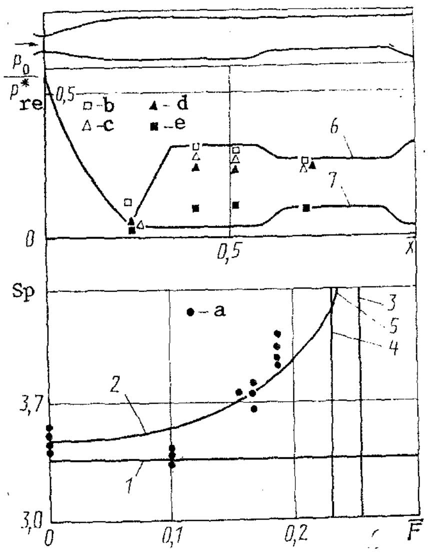

Fig. 4. Dependence of the startup pressure ratio Sp for a supersonic diffuser (theoretical Mach number M = 3 prior to the diffuser in operating conditions) on the relative neck of the diffuser – $\bar{F} = (F_{in} - F_b)/F_{in}$ – and the static-pressure distribution over the channel length at various times in startup conditions with $\bar{F}$ = 0.1: a) Sp, calculation; 1) pressure ratio in direct pressure discontinuity when M = 3; 2) by proposed method; 3) from the condition of existence of a direct pressure discontinuity prior to the diffuser; 4) experimental values; 5) by the proposed method; b) t = 12.92 sec; c) 12.96; d) 12.98; e) 13.00; calculation by the proposed method: 6) t = 14.10 sec; 7) 14.22.

The theoretical and experimental dependence of the startup pressure ratio for a supersonic diffuser with a Laval nozzle when M = 3 on the relative neck of the diffuser and the static-pressure distribution over the channel length is shown in Fig. 4 at various times in the motion of a system of pressure discontinuities for one of the diffuser profiles. The rate of increase in total pressure prior to the Laval nozzle over time in the experiments is 0.01-0.03 MPa/sec.

It is evident from analysis of Fig. 4 that, with some definite diffuser profile, there may be minimal startup pressure difference, close to the total-pressure loss in a direct pressure discontinuity with the working Mach number at the Laval-nozzle output. This arises in that disruption and localization of the return flows from the channel output to the Laval nozzle occurs with narrowing of the diffuser. Calculation of the startup pressure difference by the method here proposed (curve 1) more accurately describes the influence of the diffuser profile than one-dimensional calculation [11] (curve 2). However, the present method takes no account of disruption and localization of the breakaway flows in startup conditions.

Thus, the results obtained by the proposed method are in satisfactory agreement with the experiment in all the cases considered.

5. Conclusions

The engineering method of calculation proposed here permits the determination of the optical flow-through geometry of a supersonic diffuser from the condition of minimum total-pressure loss in the diffuser with sufficient accuracy for practical purposes, as well as the estimation of the influence of various perturbing factors on the operational efficiency of the diffuser.

NOTATION

x, y, coordinates along the channel axis and the normal; L, characteristic dimension; $\bar{x} = x/L$, dimensionless coordinate along the x axis; h, numerical-integration step along the x axis; t, time; δ, δ_T, thickness of dynamic and thermal boundary layers; δ^*, δ^{**}, δ_T^{**}, displacement, momentum-loss, and energy-loss thicknesses; $H = \delta^*/\delta^{**}$, form parameter of boundary layer; $\xi = y/\delta$, dimensionless coordinate along the y axis; F, F^*, F^{**}, cross-sectional area of channel, displacement, and momentum loss; u, projection of mean velocity onto x axis; $\omega = u/u_e$, dimensionless velocity along x axis; a, sound velocity; p, pressure; ρ, density; $\tilde{\rho} = \rho/\rho_e$, dimensionless density; T, temperature; i, enthalpy; e, internal energy; τ, tangential stress; $\tilde{\tau} - \tau/\tau_w$, dimensionless tangential stress; μ, dynamic viscosity; $\tilde{\tau}_0 = \tau_0/\kappa_{w0}$, dimensionless tangential stress in standard conditions; $M = u_e/a_e$, Mach number; ε, nonsimilarity coefficient of velocity and temperature fields; q, energy flux density on account of molecular and turbulent heat transfer; i_w^*, T_w^*, equilibrium enthalpy and temperature of wall; $\psi = i_w/i_e$, enthalpy factor; $\psi_0^* = i_{w0}^*/i_e$, adiabatic kinetic enthalpy factor; $\psi^* = i_w^*/i_e$, kinetic enthalpy factor; $\Delta\psi = \psi - \psi_0^*$, heat-transfer factor, Ψ_M, Ψ_T, Ψ_{λ_0}, Ψ_f, $\Psi_{\lambda_0\Sigma}$, Ψ_{in}, Ψ_R, Ψ_{di}, relative frictional laws for taking account of the influence of compressibility, nonisothermal conditions, the longitudinal pressure gradient as a function of the form parameters λ_0 and f, respectively, the longitudinal pressure gradient and nonsteady conditions, injection, roughness, and the pressure discontinuity, respectively; Ψ_{sr}, Ψ_{sin}, Ψ_{sdi}, Ψ_{sh}, relative heat-transfer laws for taking account of the influence of roughness, injection, pressure discontinuity, and nonsteady thermal conditions; $\Psi_\Sigma = (c_f/c_{f0})_{Re^{**}}$, $\Psi_{s\Sigma} = (St/St_0)_{Re_T^{**}}$, relative frictional and heat-transfer laws; $St = q_w/\rho_e u_e(i_w^* - i_w)$, generalized Stanton number; $c_f = 2\tau_w/\rho_e u_e^2$, frictional coefficient; $c_{f0} = 2\tau_{w0}/\rho_e u_e^2$, frictional coefficient of steady, isothermal, incompressible boundary layer at impermeable smooth plate (standard conditions); $f = (\delta^{**}/u_e)\partial u_e/\partial x$, $f_\delta = (\delta/u_e)[\partial u_e/\partial x + (1/u_e)\partial u_e/\partial t]$, $f_i = [1/(i_w^* - i_w)]\partial(i_w^* - i_w)/\partial\bar{x}$, form parameters; $Re^{**} = \rho_e u_e \delta^{**}/\mu_e$, $Re_T^{**} = \rho_e u_e \delta_T^{**}/\mu_e$, $Re_L = \rho_e u_e L/\mu_e$, Reynolds numbers; $\lambda_0 = -(\delta^{**}/\tau_{w0})\rho_e u_e(\partial u_e/\partial x)$, $\lambda_{0\Sigma} = -(\delta^{**}/\tau_{w0})[\rho_e(\partial u_e/\partial t) + \rho_e u_e(\partial u_e/\partial x)]$, form parameters; $\varphi = \xi/(2\xi + 1)$, $D_0 = (1 - \omega_0)/\sqrt{c_{f0}/2}$; j_w, mass velocity through wall surface; $\bar{j}_w = j_w/\rho_e u_e$, relative mass velocity through wall surface; c_p, specific heat at constant pressure; $m = 2n/(1 + n)$; n, exponent in power-law velocity profile; Sp, ratio of pressure in receiver to static pressure at output from supersonic diffuser at startup. Indices: e, parameters in flow core; 0, standard conditions; w, wall; re, flow parameters in receiver; cr, critical parameters; *, stagnation parameters.

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